Program and book of abstracts

LOCATION: UNIVERSITY OF AVIGNON, CAMPUS HANNAH ARENDT, LECTURE-ROOM 2E07

Program

9:00-9:20	Welcome
9:20-10:00	**M. Théra** Metric Regularity and Directional Metric
10:00-10:40	**M. Volle** On Duality for Extended Monotropic Optimization and Application to the Infinite Sum of Functions
10:40-11:10	COFFEE BREAK
11:10-11:50	**G. Crespi** Set-optimization and variational analysis: new perspectives and challenges
11:50-12:30	**D. Aussel** Existence results for variational inequalities: recent advances
12:30-14:00	LUNCH
14:00-14:20	Some words about Dinh The Luc, by A. Seeger.
14:20-15:00	**A. Daniilidis** Gradient flows and determination of convexity
15:00-15:40	**D. Gourion** Lagrangian duality in multiple objective linear programing
15:40-16:10	COFFEE BREAK
16:10-16:50	**E. Ernst** Domination property in a finite-dimensional setting : an application of a theorem by Dinh The Luc
16:50-17:30	**J.-E. Martinez-Legaz** Non-polyhedral extensions of the Frank and Wolfe Theorem

Metric Regularity and Directional Metric

Michel Théra
Université de Limoges

My aim in this talk is to make an overview on metric regularity, a concept which plays an important role in optimization and related topics and has attracted over the recent years a large number of contibutions. Then, I intend to give some new advances on directional metric regularity.

On Duality for Extended Monotropic Optimization and Application to the Infinite Sum of Functions

Michel Volle
Avignon University LMA EA 2151

Dinh The Luc
Avignon University LMA EA 2151 and Center for Informatics and Computing, VAST, Vietnam

We consider the problem of minimizing the sum of two functions, one of which is separable, on a (possibly infinite) product space. This frame covers a number of problems studied in recent works by Bertsekas (2008), Bot and Cseknek (2010), Burachik and Majeed (2013) and some others. Our focus is on characterizing the stability (strong duality) property and on establishing primal-dual optimality conditions. We apply the results to problems with the infinite sum of functions for which a direct perturbational approach is also proposed.

Set-optimization and variational analysis: new perspectives and challenges.

Giovanni Paolo Crespi
University of Insubria

Andreas Hamel
University of Bozen

Daishi Kuroiwa
Shimane University

C. S. Lalitha
University of Delhi

Set-optimization is a fast growing branch of optimization that is providing a new and fresh look to both vector optimization and set-valued analysis. Optimization problems with set-valued objective functions, i.e. with values in the power set $\mathcal{P}(\mathcal{Z})$ of a locally convex topological linear space Z, are not new in the literature. Motivated by duality for vector optimization, such set-valued optimization problems have been considered first by Corley [1], [2] and Dinh The Luc [3], [4], [5], [6]. They gained popularity after the appearance of [4] and [8], where so-called set relations are investigated and used to define minimality concepts for sets.
However $\mathcal{P}(\mathcal{Z})$ is too large an object and lacks reasonable structure which can be exploited for optimization purposes. On the other hand, additional assumptions imposed on the objective function $F : X \to \mathcal{P}(\mathcal{Z})$ often imply that the images belong to a relatively small subset of $\mathcal{P}(\mathcal{Z})$, which carries a richer algebraic and order structure. For example, C-convexity of F implies that the set $F(x) + C$ is convex for all $x \in X$. Therefore, appropriate subsets of $\mathcal{P}(\mathcal{Z})$ are used as image sets of set-valued functions.
The approach developed in [9], [10] and, more recently, summarized in [11] allows for several applications to mathematical programming and finance. In [12] and [13], by set-optimization techniques, the notion of well-posedness has been extended to set-valued optimization problems. Moreover, through generalized convexity argument, robust (vector) optimization has been studied embedding the multiobjective problem into a set-valued environment. Results on generalized convexity has also been proved in [13], [14], providing a unifying approach to the definition of quasiconvex set-valued function.
In this talk we address a big hit in optimization theory, namely the Minty Variational principle and the relations between variational inequalities and optimization problems. Since the seminal papers by Stampacchia and Minty, the so called differentiable variational inequalities provide necessary and sufficient optimality conditions for scalar optimization under mild assumptions. Continuity and convexity, mostly, plays the role of key hypothesis to prove these results. In the last decades, some generalization of the classical variational inequality has been proposed to consider non differentiable optimization problems [15]. Indeed, by means of a Dini type derivative, it has been proved that the existence of a solution to the Minty variational inequality entails some regularity on the primitive optimization problem. Some attempt to extend these results to vector optimization has been undertaken as well (see e.g. [16]). However, mainly due to the lack of a complete order in the image space, the results proved are not exactly matching those for the scalar counterpart. As for the case of set-valued objective functions, only the recent introduction

of set-optimization theory has allowed to overcome most of the difficulties that were foreseeable when dealing with set-valued objective function.
The main goal is to define new lower directional derivatives of Dini type for set-valued functions and provide necessary and sufficient conditions in terms of variational inequalities of Minty type to characterize solutions of set-valued minimization problems. Two questions arise. First, what is understood by a solution of the above problem? Secondly, how can a directional derivative, in particular a difference quotient, be defined if the image set of the function is not a linear space? The answer to the first question is a new solution concept for set-valued optimization problems proposed by F. Heyde and A. Löhne [17]. This concept subsumes classical minimality notions borrowed from vector optimization as well as the infimum/supremum in complete lattices (which are usually not present in vector optimization). The answer to the second is provided by means of residuation operations in (order) complete lattices of sets which replace the inverse addition (the difference) in linear spaces. This approach has been proposed in [18]. In [19] and [20] we extend most of the results known for scalar optimization to set-optimization. Results for vector optimization can also be obtained as an application of those in set-optimization, legitimating the latter as a desirable setting to study the former.

References

[1] Corley, H. W. "Existence and Lagrangian duality for maximizations of set-valued functions." *Journal of Optimization Theory and Applications* 54.3 (1987): 489-501.

[2] Corley, H. W. "Optimality conditions for maximizations of set-valued functions." *Journal of Optimization Theory and Applications* 58.1 (1988): 1-10.

[3] Luc D.T., *Theory of Vector Optimization*, Lecture Notes in Econom. and Math. Systems, vol.318, Springer-Verlag, Berlin, 1989

[4] Luc, D. T. (1991). Contingent derivatives of set-valued maps and applications to vector optimization. *Mathematical Programming*, 50, 99-111.

[5] Khanh, P. Q., & Luc, D. T. (2008). Stability of solutions in parametric variational relation problems. *Set-Valued Analysis*, 16(7-8), 1015-1035.

[6] Luc, D. T. (2008). An abstract problem in variational analysis. *Journal of Optimization Theory and Applications*, 138(1), 65.

[7] Kuroiwa, D. Tanaka, T. and Truong, X.D.H. "On cone convexity of set-valued maps." *Nonlinear Analysis: Theory, Methods & Applications* 30.3 (1997): 1487-1496.

[8] Kuroiwa, D. "On Natural Criteria in Set-Valued Optimization (Dynamic Decision Systems under Uncertain Environments)." (1998).

[9] Hamel, A. Variational principles on metric and uniform spaces. Diss. Halle (Saale), Univ., Habil.-Schr., 2005.

[10] Hamel, A. "A duality theory for set-valued functions I: Fenchel conjugation theory." *Set-Valued and Variational Analysis* 17.2 (2009): 153-182.

[11] *Set Optimization and Applications - The State of the Art, From Set Relations to Set-Valued Risk Measures*, A. H. Hamel, F. Heyde, A. Löhne, B. Rudloff, C. Schrage eds., Springer Proceedings in Mathematics & Statistics, Springer-Verlag Berlin Heidelberg, Berlin 2015.

[12] Crespi, G.P., Kuroiwa, D. and Rocca M. "Convexity and global well-posedness in set-optimization" *Taiwanese Journal of Mathematics* 18.6 (2014): pp-1897.

[13] Crespi, G.P., Kuroiwa, D. and Rocca, M. "Quasiconvexity of set-valued maps assures well-posedness of robust vector optimization." *Annals of Operations Research* (2015): 1-16.

[14] Crespi, G. P., Dhingra, M., & Lalitha, C. S. (2018). Pointwise and global well-posedness in set optimization: a direct approach. *Annals of Operations Research*, 269(1-2), 149-166.

[15] Crespi, G. P., Ginchev, I. and Rocca, M. "Minty Variational Inequalities, Increase-Along-Rays Property and Optimization." *Journal of optimization theory and applications* 123.3 (2004): 479-496.

[16] Crespi, G.P., Ginchev, I. and Rocca, M. "Some remarks on the Minty vector variational principle." *Journal of mathematical analysis and applications* 345.1 (2008): 165-175.

[17] Löhne, A. *Vector optimization with infimum and supremum.* Springer Science & Business Media, 2011.

[18] Hamel, A.H. and Schrage, C. Notes about extended real-and set-valued functions, *Journal of Convex Analysis.* 19(2) (2012).

[19] Crespi, G.P., Hamel, A.H. and Schrage, C. "A Minty variational principle for set optimization." *Journal of Mathematical Analysis and Applications* 423.1 (2015): 770-796.

[20] Crespi, G.P., Rocca, M. and Schrage, C. "Variational inequalities characterizing weak minimality in set optimization." *Journal of Optimization Theory and Applications* (2014): 1-21.

Existence results for variational inequalities: recent advances

Didier Aussel
University of Perpignan, Lab. PROMES UPR CNRS 8521

Our aim in this talk is to make an (somehow personal) overview of the evolution of the existence-type results that have been developed in the last decades. Indeed, motivated by new fields of applications, several improvements have been proposed. Thus we would like to present a serie of applications/new results for which we will see that old/classical techniques are still of main use.

Gradient flows and determination of convexity

Aris Daniilidis
Universidad de Chile

Tahar Boulmezaoud
Université de Versailles St-Quentin-en-Yvelines

Philippe Cieutat
Université de Versailles St-Quentin-en-Yvelines

We disclose an interesting connection between the gradient flow of a C^2-function f and strongly evanescent orbits of the second order gradient system defined by the square-norm of ∇f, under an adequate convexity assumption. As a consequence, we obtain the following surprising result for two C^2, convex and bounded from below functions f, g defined on a Hilbert space $\mathcal{H}$: If for all $x \in \mathcal{H}$ we have $||\nabla f(x)|| = ||\nabla g(x)||$, then $f = g + k$, for some constant $k \in \mathbb{R}$.

Lagrangian duality in multiple objective linear programing

D. Gourion
Avignon University LMA EA 2151

D.T. Luc
Avignon University LMA EA 2151 and Center for Informatics and Computing, VAST, Vietnam

The purpose of the present work is to apply the method of Lagrangian functions to linear multiple objective programming problems. We consider the multiple objective linear program in standard form (LVP)

$$\begin{aligned} \text{Maximize} \quad & Cx \\ \text{subject to} \quad & Ax = b \\ & x \geqq 0 \end{aligned}$$

where C is a real $k \times n$-matrix, A is a real $m \times n$-matrix and b is a real m-vector.
The Lagrangian function associated with the problem (LVP) is defined to be a function of two variables: x from $\mathbb{R}^n_+$ and Y from the space of $k \times m$-matrices $L(\mathbb{R}^m, \mathbb{R}^k)$ given by

$$L(x, Y) = Cx + Yb - YAx.$$

We study saddle points of the vector Lagrangian function: we introduce three concepts of saddle points and establish their characterizations by solving suitable systems of equalities and inequalities. We deduce dual programs and prove a relationship between saddle points and dual solutions, which enables us to obtain an explicit expression of the scalarizing set of a given saddle point in terms of normal vectors to the value set of the problem. Finally, we present an algorithm to compute saddle points associated with non-degenerate vertices and the corresponding scalarizing sets.

Domination property in a finite-dimensional setting : an application of a theorem by Dinh The Luc

Emil Ernst
Aix-Marseille Université, France

A standard result in the existence of cone-minimal points is Hartley's Theorem (1978) saying that, in $\mathbb{R}^n$ ordered by the means of a convex and pointed (but not necessarily closed) cone C, any compact set A has the domination property, meaning that for any point $x \in A$, there is some cone-minimal point $y \in A$ such that $y \leq x$.
Hartley's problem asks to characterize the sets $A \subset \mathbb{R}^n$ possessing the domination property with respect to any convex cone C.
Many authors have studied the existence of minimal points in order to relax the requirement of compactness on A. In every case stronger conditions have to be imposed on the cone : to take one example out of many, Bitran and Magnati (1979) prove the domination property for closed sets A which do not posses common directions of recession with the opposite of the closure of C, but only if the cone C is strictly supported (that is contained - except the origin - in an open half-space). Hartley's problem however asks for domination results valid for any finite dimensional ordering cone.
Our communication shows that significant progress may be made in the study of Hartley's problem by using Dinh The Luc's Theorem (1989). An important step in our analysis is the characterization of the sets $A \subset \mathbb{R}^n$ with the following property : for any ordering cone C, and any point $x \in A$, then the section $(x - C) \cap A$ is C-complete.

Non-polyhedral extensions of the Frank and Wolfe Theorem

Juan Enrique Martínez-Legaz
Universitat Autònoma de Barcelona

Dominikus Noll
Université de Toulouse

Wilfredo Sosa
Universidade Católica de Brasília

In 1956 Marguerite Frank and Paul Wolfe proved that a quadratic function which is bounded below on a polyhedron P attains its infimum on P. In this work we search for larger classes of sets with this Frank-and-Wolfe property. We establish the existence of non-polyhedral Frank-and-Wolfe sets, obtain internal characterizations by way of asymptotic properties, and investigate stability of the Frank-and-Wolfe class under various operations.